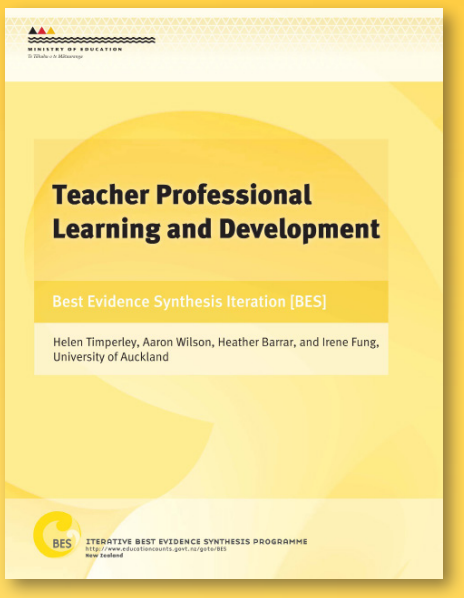
Use mathematical tools to explore students' thinking about mathematics

This is one of a series of cases that illustrate the findings of the best evidence syntheses (BESs). Each is designed to support the professional learning of educators, leaders and policy makers.



BES cases: Insight into what works

The best evidence syntheses (BESs) bring together research evidence about 'what works' for diverse (all) learners in education. Recent BESs each include a number of cases that describe actual examples of professional practice and then analyse the findings. These cases support educators to grasp the big ideas behind effective practice at the same time as they provide vivid insight into their application.

Building as they do on the work of researchers and educators, the cases are trustworthy resources for professional learning.

Using the BES cases

The BES cases overview provides a brief introduction to each of the cases. It is designed to help you quickly decide which case or cases could be helpful in terms of your particular improvement priorities.

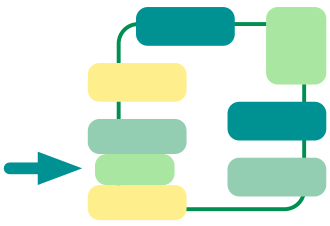
Use the cases with colleagues as catalysts for reflecting on your own professional practice and as starting points for delving into other sources of information, including related sections of the BESs. To request copies of the source studies, use the Research Behind the BES link on the BES website.

The conditions for effective professional learning are described in the Teacher Professional Learning and Development BES and condensed into the ten principles found in the associated International Academy of Education summary (Timperley, 2008).

Note that, for the purpose of this series, the cases have been re-titled to more accurately signal their potential usefulness.

Responsiveness to diverse (all) learners

Use the BES cases and the appropriate curriculum documents to design a response that will improve student outcomes



The different BESs consistently find that any educational improvement initiative needs to be responsive to the diverse learners in the specific context. Use the inquiry and knowledge-building cycle tool to design a collaborative approach to improvement that is genuinely responsive to your learners

Use mathematical tools to explore students' thinking about mathematics

This case explains how the effective use of three Numeracy Project tools (the number framework, diagnostic interview, and teaching model) enables teachers to design rich mathematical tasks for diverse (all) students. The tools support teachers to explore students' thinking about mathematics, their own expectations of students, and their beliefs about teaching mathematics.

Recent international studies show that in-school variation in mathematical achievement is greater than the variation between schools. This case describes some of the processes schools can use to build teachers' mathematical knowledge and pedagogical practices so that they can employ the tools effectively to improve mathematical outcomes for all students and.

See also BES Exemplar 1: *Developing communities of mathematical inquiry and BES Case 26: Strengthen teacher pedagogical knowledge.*

Using evidence of student thinking to inform pedagogy

The Numeracy Development Project (NDP) is a national project that has (at the time of writing) involved approximately 690,000 students and 23,000 teachers. Initially, as part of a planned roll-out to all schools, individual schools were invited to participate. It is anticipated that eventually all teachers at the target year levels will have been involved. There is an expectation that senior management will participate in the professional development but this does not always happen. All schools appoint a lead teacher but the role is variously interpreted.

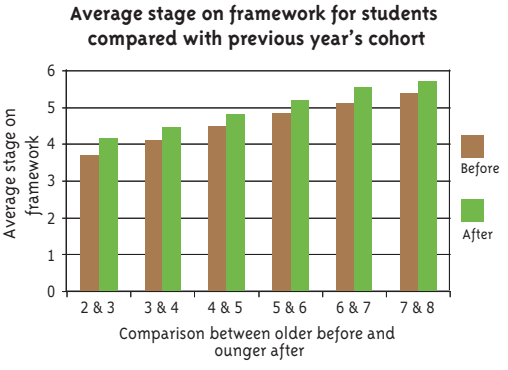
Each teacher is provided with 13.2 hours of professional development in the first year. They participate in six to eight workshops of 2.5 hours' duration and receive at least three in-class observations followed by feedback with the facilitator. Facilitators make decisions about the timing and form of the professional development in consultation with the lead teacher(s). In the second year, teachers participate in up to 8 further hours of professional development.

The professional development emphasises the development of strategic thinking and mathematical knowledge through discussion of multiple solutions to problems. This contrasts with the prevailing models of practice, which typically stressed completion of written mathematical exercises following textbook progressions.

The numeracy project is a system-wide initiative to raise students' achievement in mathematics and develop teacher knowledge of teaching mathematics. It came about in response to the recommendations of the Mathematics and Science Taskforce (1997).

An important component of the project is the Number Framework, which consists of a sequence of global stages describing the mental processes students use to solve problems with numbers. At lower stages on the framework the steps are smaller than at upper stages, so progress appears greater. Prior to PD, students in the early years of school progress approximately a stage per year on addition/subtraction. Progress for students in the middle years of primary school is about half a stage per year. In years 7–8, average progress on addition/subtraction is about a fifth of a stage per year.

Since the project's beginnings in 2000, the students of the teachers involved have consistently shown substantive progress, measured against the Number Framework. The overall effect size is 0.34, with average effect sizes of 0.39 for multiplication/division (see graph at right, showing 2006 data) and proportion/ratio. The effect size for addition/subtraction is smaller at 0.24. Groups with an average effect size above the overall average include Pasifika students (0.40), Māori (0.35), and students from low-decile schools (0.38). These effect sizes are based on 2005 and 2006 data and compare students after a year on the project with slightly older students (those in the next year level) before the project started. (For example, year 2 NDP students at the end of 2005 were compared with year 3 pre-NDP students at the start of 2005.) The project has evolved as ongoing evaluations have informed its implementation. A network of stakeholder and participating groups has contributed to the review process.



PD/PL activities

Creating dissonance

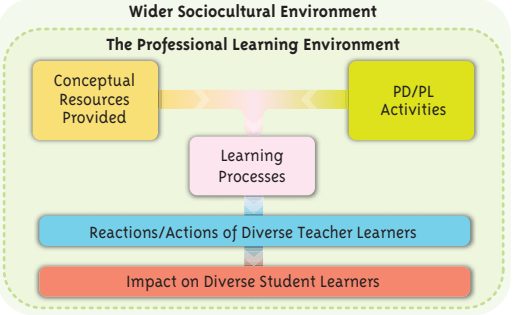
In the early stages of the PD, the focus was typically on challenging teachers' previously held assumptions about student learning of mathematics. They learned to use a diagnostic interview to determine the level at which each student was thinking, measured against progressions on a number framework. As they interviewed their students, teachers were often surprised to find that their previous judgments were inaccurate. This challenged them to examine their current practice, expectations of students, and beliefs about teaching mathematics. Their discomfort with their current practice provided the rationale and motivation for them to engage with a different model of teaching.

Teacher

… some of the results really blew me away, it was the processes, what they knew … I had children who I had expected to do better, but they didn't, and I had some children who I expected to be at a certain level and [they] ended up being quite high up in terms of their cognitive processes, but they didn't give me the answer, I had just assumed, oh no, you're this level … It got rid of all my preconceived ideas.

Modelling new practice

As teachers are adult learners with considerable practical experience and existing professional knowledge, it was necessary to give them evidence that alternative practices were relevant and effective. To provide such evidence, providers modelled recommended teaching approaches in individual teachers' classrooms with their own students. They were often able to elicit explanations from students in ways that the teachers had not thought possible. Teachers were motivated to change as their beliefs about what students could learn were challenged and as they saw convincing alternative practice modelled.



Provider

It takes time, but a very effective thing that this contract has done is to be in [the teacher's] classroom with their children, modelling what it looks like and what's possible. We model with your kids and they say, "I never knew that kid knew that", or "This is where this is coming from." I think the effective thing has been going in there facing their classroom and the fact that you keep on coming back … You are able to talk with the teacher about underlying concepts.

Developing content knowledge

Underpinning the professional development was an assumption that effective teaching practice is informed by strong content knowledge. Different theories of motivation suggest different approaches to addressing this issue. Workshops on the number framework and the diagnostic interview were a mandatory feature of the project. Providers met the need for further content knowledge in separate workshops or, following a more integrated approach, addressed them as gaps became evident in the course of professional development activities.

The number framework describes increasingly sophisticated stages of strategy and mathematical knowledge. A graduated teaching model provides teaching approaches for each stage of the framework. This combination of learning resources gave teachers a theoretical framework for thinking about numeracy progressions, and appropriate activities for use with students. Facilitator and video demonstrations plus teachers' own use of the diagnostic interview helped them unpack the stages of mathematical thinking exemplified on the framework.

Refining new practice

Teachers received detailed instructional activities and resources, but how they were used varied depending on the professional development provider. They were sometimes used as a resource for students and sometimes as a resource to support teachers' development of theoretical ideas.

The theoretical approach to facilitation was implicit rather than explicit. For this reason, on-the-ground facilitation approaches varied considerably from one Ministry of Education contract to another. This allowed providers to be responsive to the contexts in which they worked and to the specific needs of their groups of teachers. It also supported the development of understandings, beliefs, and goals that were shared by providers and teachers.

Why did this work?

The project had a clear goal of raising student achievement and developing teacher knowledge. Teachers were motivated to change when they saw evidence of discrepancies between perception and reality, relating to students' mathematical understanding and capacity to learn, and as they were provided with theoretical frameworks on which to base new practice.

The content focused on three related tools: the number framework, the diagnostic interview, and the teaching model. The number framework described increasingly complex stages of mathematical thinking. The diagnostic interview provided a means to establish where students' thinking fitted on the framework. The graduated teaching model equipped teachers to introduce their students to increasingly abstract representations of mathematical ideas. This close connection between assessment of student understanding and teaching activities was a major factor in achieving changes in teacher practice.

Teacher responses to PD/PL

Learning resources

The professional development was deliberately situated in syndicate or whole-school groupings to foster the development of professional learning communities. Providers encouraged senior management and lead teachers to facilitate teacher discussion of issues related to the professional development and to examine student progress. One lead teacher described the process of supporting teachers to change their practice in this way:

> *Getting staff through that realisation that their professional judgment, their listening to the children, their looking at the child … to say, 'OK, right, I'm quite confident that they've got that understanding, they're using that strategy, they're ready to move on.'*
>
> Lead teacher

Developing new understandings

Teachers contextualised the framework in terms of their own practice by using it and interviewing students to determine their mathematical understandings. The close connection between assessment of understanding and teaching activities was a major feature of this project. By linking content knowledge and pedagogy, teachers were better able to understand the level that their students were working at and what their next teaching steps should be.

Provider

> *Success for most of [the teachers] has been undeniable when they see the results, and they can tell you about their kids far more than they ever could. It has been about building teacher knowledge, their own, personal knowledge in maths.*

Teachers viewed their professional learning as adding to their repertoire of teaching strategies and they were often surprised by the degree of change their practice underwent. Repeated opportunities to develop their own understandings, link these to the framework, observe the provider modelling practice with their own students, and discuss underlying concepts and theories allowed teachers to connect the theoretical framework, the teaching model, and the teaching activities, and to gradually embed new practice in their own contexts.

> *[The way I teach] has changed completely. I am shocked when I look back to my teaching prior to ANP. My teaching is targeted more closely to individual needs … the framework is at the centre of my teaching now. Teaching strategies as well as knowledge were new to me.*
>
> Teacher

Teacher reactions

Major learning processes

Interviews, surveys, and case study observations of teaching practice showed that most teachers found the process challenged their current assumptions about what aspects of mathematics should be taught, how to teach it, and how students learn. Many teachers reported that engagement with the professional development process led to reconstructing previously held views. Teachers were able to articulate the ways in which theory underpinned their practice and chose activities in terms of the teaching purpose, informed by the mathematical framework.

Such substantive differences in approaches to teaching meant that implementation of selected aspects of the new approach was likely to create a misalignment between new and existing practice and create substantial cognitive dissonance. In order to achieve the accelerated stage gains, the most common reaction of the professional development was active engagement and application of new theory and practice while participating in the professional development. The extent to which the new practices are sustained is not known, although the development of the lead teacher model may assist in maintaining gains.

How did the teachers make this work?

Teachers came to understand how their practice needed to be aligned with the learning needs of their students. Using the diagnostic interview, they came to better understand their students' mathematical thinking. The framework helped them to determine next learning steps for their students and to be aware of developmental progressions in student thinking.

Through multiple and ongoing opportunities to apply their new learning in the context of their own classrooms, teachers were able to translate their new learning into practice. Another benefit of applying new practice was that teachers' own mathematical knowledge was deepened. As they better understood what their students could and should be learning, their commitment to using new pedagogical methods was strengthened. Student outcomes confirmed that their shifts in practice had been effective.

How this case links to the synthesis

Professional learning and mathematics

6.2.1.5 External expertise

6.2.1.8 Prevailing discourses and models of practice

6.2.2.2.1 Teachers' knowledge of how students learn mathematics

6.2.2.3 Assessment

6.2.3.1 Activities to create dissonance/problematise existing practice

6.2.3.2 Activities that helped teachers translate theory into practice

Overview 6.3: Activities constructed to promote the professional learning

6.2.3.5 Examining student outcomes and understandings

Topical issues

10.1 Issue 1: Multiple roles of assessment in promoting teacher learning

10.1.4.3 Where to next?

10.3 Issue 3: Teachers' existing theories

10.3.5 Sequence of change

Reflective questions

Teachers were able to make substantial shifts in both their practice and the achievement of their students.

- What features of this project enabled teachers to use information about student learning to make these improvements?
- What aspects of the professional development motivated teachers to change their practice?

Sources

Higgins, J. (2004). *An evaluation of the Advanced Numeracy Project 2003* (Report to the Ministry of Education).

Young-Loveridge, J. (2005). Patterns of performance and progress on the numeracy projects: Analysis of 2004 data. In J. Higgins & K. Irwin & G. Thomas & T. Trinick & J. Young-Loveridge (Eds.), *Findings from the New Zealand Numeracy Development Project 2004* (pp. 5-20, 115-127). Wellington, NZ: NZ Ministry of Education.