

**A SOCIOCULTURAL ANALYSIS OF CHILDREN'S
PARTICIPATION IN A MATHEMATICAL TASK**

RESEARCH REPORT OF

A PROBE STUDY

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EXECUTIVE SUMMARY

This report outlines the findings from a research project that examined videotapes of six groups of four children participating in the NEMP *Farmyard Race* task. The videos were analysed using a sociocultural framework. The findings from the analysis highlight that children's participation in group mathematical tasks has intellectual, physical-spatial, material and social-relational dimensions: The findings suggest:

- a context can be motivating but difficulties can arise when children are expected to engage with a context whilst simultaneously ignoring factors that would be pertinent in a “real-life” situation;
- understanding and expression of mathematical ideas is bound up with language;
- the meaning of ordinal words can be ambiguous in relation to a particular context;
- children make decisions by deferring to an authoritative member, by democratic means and or by the aggregation of information;
- children find it difficult to aggregate information;
- manipulatives can serve as a means of organising a task, a problem space for its solution and a final product that is the outcome of the solution process;
- momentary configurations of manipulatives and children's talk interlock to form and shape multimodal communication;
- manipulatives can serve as focal artefacts for collaborative problem solving;
- the physical-spatial arrangement of children in relation to a task problem space shapes access to collaborative work;
- group activities rely on children having and deploying a range of social practices; .
- a child's social standing and skills influence access to talk and materials;
- the child/ children who assume leadership responsibility impact on group goals and achievements;
- children's purposes shift and take form as they interact about a task;
- children's mathematical goals are interwoven with the context and artefacts of a task.

A sociocultural interpretation of children's participation in group mathematics assessment tasks highlights several aspects. These are:

- the product of a group deliberation provides a restrictive view of what children know and can do;
- information about the process of reaching a solution provides more insight into children's thinking;
- tasks do not always constrain children's thinking in ways that lead them to being able to accomplish the task as it was intended;
- a contrived context relies on children's appreciation of the nature of school mathematical tasks;
- a contextualised task can introduce language demands to do with the boundary between everyday language and experience and the particular ways language is used in the register of mathematics;
- children's lack of familiarity with a task structure may obfuscate what they know and can do;
- a task needs to demand that all children contribute to its conceptual and practical outcome to evoke genuine cooperation.

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SECTION 1: INTRODUCTION

Recent research suggests that pencil and paper tests may not reflect all the mathematics that students know and can do (Boaler, 2003; Clarke, 2003; Harlow and Jones, 2003; Schoultz, Saljo and Wyndhamn, 2001). Non-written group tasks offer another lens into children's mathematical learning; a lens that generates multimodal information about the ways children work together to arrive at a solution to a task. Video is a useful tool in this context. It can capture dialogue, actions and interactions to help make more of the complexity of children's engagement with the task available for later analysis. However, any analysis of group tasks poses challenges in terms of how and what information to attend to. It requires the development of analytical frameworks that acknowledge the complexity and richness of children's mathematical activity.

One of the achievement aims in the *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992) document is that children 'become effective participants in problem solving teams, learning to express ideas and to listen and respond to the ideas of others' (Ministry of Education, 1992, page 9). This recommendation and other suggestions such as having problems set in real life contexts and children being encouraged to use apparatus are consistent with social or situated views of learning. In this study we utilised the potential of a sociocultural framework to make sense of children's engagement with a mathematical task.

The first part of this report provides a brief review of the literature pertinent to children's engagement in a contextualised mathematical task. The second part of the report presents an analysis of the videotapes of six groups of four children as they worked on the NEMP *Farmyard Race* task. Finally, implications for the teachers and assessment are discussed.

SECTION 2: SETTING UP A SOCIOCULTURAL FRAMEWORK FOR THE ANALYSIS OF CHILDREN'S PARTICIPATION IN MATHEMATICAL TASKS

2.1 Connecting to sociocultural views of learning

Situated and sociocultural views locate learning in social and cultural activity. They focus attention on the interactions between people, the tasks they seek to accomplish and the setting they are in. Classroom researchers are turning to social and situated views of learning to help them describe and enhance children's mathematical learning (Cobb, 2000; Lerman, 2000). From a sociocultural perspective knowledge and understanding of mathematics are not seen as attributes that an individual possesses but as social practices of sense-making and problem solving using mathematical representations, concepts and methods as resources (Boaler, 1999). The tasks and practices in which children participate constitute the mathematics they come to learn and know (Boaler and Greeno, 2000; Wenger, 1998; Wertsch, 1991).

Assessment activities are a special kind of situation in which knowledge of how to do mathematics matters. When children take a test they show how well they can participate in the kinds of interaction that the particular test affords (O'Connor, 1989). Any analysis of children's knowledge and understanding of mathematics needs to take into account the nature of the assessment task, the language the children draw on and use, the patterns of interaction that develop, and the use made of tools.

2.2 Context and children's responses to mathematical tasks

Mathematical tasks are often posed in everyday contexts that are intended to motivate, illustrate potential applications, act as a source of opportunities for mathematical reasoning and thinking, and to anchor student understanding (Meyer, Dekker and Querelle, 2001). However, as Boaler (1993) points out, 'no one task context can offer a universal application which is familiar and, more importantly, meaningful for all students' (page 14). Setting a mathematical task within a 'real' context blurs the boundary between school mathematics and everyday concerns. Children need to understand the problem as a school mathematics task, to identify and undertake the mathematics necessary to solve the task, and interpret what the answer means in relation to the context. They need to read and comprehend the written text and have to judge very finely how much realistic everyday knowledge to use (Cooper, 1998). It is apparent that tasks presented in context pose a number of literacy, linguistic and interpretive demands (Boaler, 1993; Cooper and Dunne, 1998).

Children need time to think about the context and to consider what aspects might be relevant to the problem in hand, make sense of it in relation to the problem that is posed and to distinguish where and when it is appropriate to use their everyday knowledge (Cooper, 1996; Stern, 2000; Sullivan, Zevenbergen and Mousley, 2003; Taylor and Biddulph, 1994). Familiarity with a context is not necessarily beneficial. Children can draw on their experiences in ways that can be unhelpful to their understanding the underlying mathematical intent of the task (Boaler, 1993). Conversely, children may not draw on their everyday experience when it is appropriate to do so. In New Zealand, Anthony and Walshaw (2003) found the knowledge children brought to the context played a role in their solution processes in mathematical assessment tasks.

2.3 Language and children's responses to mathematical tasks

Language is bound up with learning. It provides the medium through which communication of ideas is made possible and the negotiation of ideas is achieved. Increasingly, mathematics educators are recognising that mathematics uses language in particular ways (Zevenbergen, 2000). Children trying to make sense of a task need to attend to this 'register of mathematics' to appreciate its mathematical intent (Zevenbergen, 2000). For instance, the word 'volume' has an everyday and a mathematical meaning. 'Square' has a different meaning in geometry than in number and the word 'before' signals that a task involves consideration of order. A word or phrase that has an accepted meaning in everyday circumstances can be ambiguous in the context of a mathematical problem (Moschkovich, 2002; Pimm, 1987; van Oers, 2001).

2.4 Children's use of manipulatives in mathematical tasks

Children can use manipulatives as a tool to help them explore and characterise ideas in ways that are not possible without it (Ministry of Education, 1992). Manipulatives can also serve to focus collective activity on a task and help children to describe and communicate their mathematical ideas to others (Meira, 1996). They can help represent abstract ideas and provide opportunities for children to explore different possible solutions and solution processes (Moyer and Jones, 2004). Children may use apparatus in ways that can lead to idiosyncratic conceptions of a task (Kanes, 1998) and find it difficult to use manipulatives to build up a solution to a problem (Hart, Johnson, Brown, Dickson and Clarkson, 1989). To make effective use of a manipulative, children have to conceive it as a mathematical idea at the same time as they treat it as an object. Research suggests too, that children may focus on the manipulative per se rather than as an alternative form of expression (Moyer, 2001).

2.5 Participatory patterns in mathematical activity

For children as students in classrooms, participation in mathematical activity is shaped by the particular social and sociomathematical norms that have been established between them and their teachers. Social norms comprise of expectations and obligations about roles that can be assumed by teachers and students when they participate in mathematics lessons (Cobb, Gravemeijer, Yackel, McClain and Whitenack, 1997). These include taken for granted patterns of participation such as turn taking and listening to other children (Greeno, 1997). Sociomathematical norms frame what constitutes a valid process for reaching a solution, what constitutes an acceptable solution and who has the authority to make this judgement (Yackel, Cobb et al., 1990). For instance, children may have come to believe there is one right answer known by the teacher for a mathematics problem or they may expect to discuss and debate ideas (Blumenfeld and Mergendoller, 1992). The negotiation of mathematical meaning occurs when children explore and consider each other's reasoning and view points (Goos, Galbraith and Renshaw, 1996). This negotiation requires a shared understanding of the task and social decision-making skills. Children working on group assessment tasks may struggle with this (Harris, 2003). The physical arrangements of the group setting for children's task engagement influence the ways in which children interact (Roth, McGinn, Woszczyzna and Boutonne, 1999). A child's spatial location can influence their access to focal artefacts in ways that shape their opportunities to contribute to ongoing conversation and a shared representation of the solution (Roth and Roychoudhury, 1994).

2.6 Children's responses to the mathematical demands of tasks

There is worldwide interest in children's understanding of mathematics, more particularly their understanding of number, including ideas about cardinal number, fractional number, place value and

decimals. Texts about children's mathematical development include sections that explain what ordinal numbers are without elaborating possible learning pathways for children's development of ordinal concepts. Miller, Major, Shu and Zhang (2000) argue that ordinal ideas have special attributes that can make them conceptually difficult to appreciate. Ordinal referents are plurismatic or indexical (Voigt, 1994). Their meaning and implication can be significantly different depending on the context in which they are used. To make sense of word or phrase it is necessary to use background knowledge that forms a context for interpretation. Depending on children's background understanding an ordinal referent may be experienced as ambiguous or factual.

2. 7 Children's purposes while doing a task

Children need to appreciate the mathematical intent of a task if their actions are to align with those of the teacher or assessor. Research has highlighted that this can be problematic; children do not always appreciate their teacher's goals for and purposes of their engagement with a particular mathematical task (Voigt, 1994). Students pursue a range of academic and social goals. For instance, Dweck (1986) noted that children pursue learning and or performance goals that are context dependent. Testing has been attributed with promoting performance goals whereby students concentrate on obtaining the 'right' answer and gaining recognition from peers and or the teacher.

SECTION 3: RESEARCH QUESTIONS AND APPROACH

3.1 The parameters of the study

The goal of this study was to investigate the utility of sociocultural theory as a framework for making sense of children's participation in a group mathematical task undertaken by children as part of the NEMP mathematics study in 1997. The research questions were:

- What does a sociocultural lens illuminate in children's mathematical activity?
- What patterns of participation are afforded by the mathematical task?
- What language use and mathematical thinking are afforded by the task?

3.2 The *Farmyard Race* task

We decided to focus on the NEMP *Farmyard Race* task after viewing of a small sample of videos of children working on a range of NEMP mathematical tasks.

The *Farmyard Race* task is a cooperative logic problem (Erickson, 1989). The race contestants are eight common farm animals: a cow, a dog, a sheep, a goat, two horses, a pig and a piglet. To introduce the task to the children, each NEMP administrator positioned a placement mat on to the table between the children. He

or she outlined the purpose the task as *your team is going to put the animals in the order that they came in the race*. He or she then placed eight plastic animals, one-by-one, onto the table beside the placement mat, naming each animal in turn. Next the administrator gave each child one of the four clue cards with information pertinent to the finishing order on it. The children were instructed that their task was to find the finishing order of the animals and were directed to read their clue card. The children were left to work on the task. When the children and/or the administrator considered the task completed the administrator recorded the finishing order of the animals. Then the administrator instructed the group to *go back and check one more time to make sure that what you've done fits with all of the clues*. The children were again left to work on the task. Once they had completed this, the administrator recorded the second series of results.

To solve the task the children needed to combine the information from each of the clue cards to decide the precise order in which each of the eight animals finished the race. That is, they needed to work as a team to aggregate clue information to do with order represented by ordinal numbers such as “third” and spatial markers such as “before”, “after”, “between” and “followed”. They needed to make inferences about the consequences of information that would impact on the finishing order such as *stopping to eat*.

The finishing positions of the dog and the piglet were determined by the clues: *Dog was **third** to cross the finish line* and *Piglet **finished second to last*** (bold added). One clue specified the relationship between the mother pig and her piglet without prescribing who was leading: *2 animals **came between** mother pig and her piglet*. One clue intimated that the goat finished immediately after the mother pig: *Goat **wanted to bite mother pig's tail** as he **followed her across the finish line***. Another clue specified that sheep and piglet ran together and that piglet tripped right at the end of the race, implying that sheep crossed the finish line before piglet: *Sheep and piglet **ran together until the end when piglet tripped***. Two clues gave information about the finishing order of the white horse: *The white horse finished **before** the goat* and *The white horse saw **four legs beat him home***. It is implied in the latter clue that as only four legs crossed the finish line before the white horse then just one of the animals in the race could be in front of it. The remaining clue detailed the brown horse's position up until a certain point, and the children had to infer the consequences of the brown horse stopping to eat: *The brown horse **led the pack until** he stopped to eat*. None of the eight clues explicitly specified the relationship of the cow to the other animals in the race. The correct order for the animals is cow, white horse, dog, mother pig, goat, sheep, piglet and brown horse.

3.3 The research analysis

For this study we analysed video recordings of how six groups of four year 4 children went about completing the NEMP animal race task. The groups were selected for us by the NEMP team as representing children from a range of backgrounds; rural / urban and high/ medium and low decile schools. We decided to focus on the Farmyard Race task after viewing of a small sample of videos of children working on a range of tasks.

The video of each group was viewed a number of times. Our use of a sociocultural analysis focused our attention on the:

- ways the children made sense of the context of the task;
- children's patterns of participation (how/if and when the children drew on ways of working together to collaborate to complete the task);
- children's use of manipulatives;

SECTION 4: THE TASK AND CHILDREN'S RESPONSE TO IT

This section details the children's responses to the farmyard race task in terms of their overall approach to completing it, their responses to the context, the language used and their responses to the mathematical ideas

4.1 Group 1 and the *Farmyard Race*

Group 1 was comprised of three boys and a girl. The children were seated around the short end of a rectangular table: Cam on the left of the video camera, James and Andrew directly in front of it and Katy to the right. The administrator placed the placement mat and plastic animals in front of the boys. All four children had easy access to the animals and the mat.

Doing the task

From the start the children were committed to working together to solve the problem. They reiterated points they had made, they re-voiced statements made by others and made links to earlier propositions. Although there was some dissonance between consecutive individual contributions the dialogue moved steadily towards a group solution. Any differences were resolved through social decision-making. The children worked independently to solve the problem for seven and a half minutes then the administrator asked if they were satisfied with their solution. She recorded this and then asked them to check their solution. The children then revisited the problem and after four and a half minutes they decided upon another finishing order that the administrator recorded. Their first attempt placed the goat first, white horse second, dog third, pig fourth, cow fifth, sheep sixth, piglet seventh and the brown horse eighth. Their second version from first to eighth was sheep, white horse, dog, pig, goat, cow, piglet and brown horse.

The children and the context

The children's past experience of the speed of cows and horses was influential in their deliberations about the finishing order of the animals. No clue mentioned the cow and the children drew upon their experience as they searched for a rationale for its position. James mentioned the cow first.

- James: *This one [the cow] is really slow, so it must go there.* (James put the cow near the back of the recording grid.)
- Katy: *No they're not [slow] because ... we have bulls next door.*
- Andrew: *It might of [come last], but we don't know that do we James?*

Cam: *The cow could have been bitten by the dog. It could have gone racing forward at the speed of light. You never know.*

James utilised his everyday experience to assert that cows were slow and hence the cow should be towards the back of the field. Katy offered an alternative view that the *two bulls next door could run really fast*. Andrew seemed to argue against the use of everyday knowledge when he asserted they did not *know* that the cow was slow and hence would come last, although he conceded that it might. Cam offered a scenario for why the cow might have run fast.

One child, Andrew, questioned whether it was appropriate to draw upon everyday experiences to solve the task through his questioning of the authenticity of it. Andrew introduced the question of whether the task was authentic and reiterated the possibility it might not be whenever the others sought to use their everyday knowledge of animals to justify a placement.

James: *And this one is really slow, so it must go there.*

Katy: *Which one?*

Andrew: *It could just be a fake story, it could just be a made up story.*

At one stage Andrew held up the cow and asserted, *It's not as if this guy's going to start running is it? It's plastic*. Here he seemed to call into question the whole context and format of the task. None of the other children acknowledged his comment. It appeared that only Andrew was aware that the task provided a particular setting for engaging with mathematical ideas although he too offered suggestions for placement based on everyday experiences. The other children made no comments that indicated they distinguished the boundary of school mathematical tasks, so group actions and interactions fluctuated between a focus on everyday knowledge and using the information expressed in the clues.

The children and the language and mathematical ideas embedded in the task

The language of the task posed some challenges. The clue 'The brown horse led the pack until he stopped to eat' was a source of confusion because of the possibilities of the meaning of "*until*" in relation to when and where it stopped to eat. Katy asserted: *Horses are one of the fastest things in the world*. All four children agreed that the speed of horses is fast, therefore one could be expected to win a fair race. After further consideration of the clues, they recognised that the word 'until' signalled the brown horse had lost its leading position when it stopped to eat. From here it was decided that the brown horse could not have won the race. The children then began to try and reconcile their knowledge of the speed of horses with the clues. They deliberated whether the clue statement could accommodate the idea that the brown horse stopped to eat after the race and therefore it could win this event.

Cam: *If he stopped to eat, how could he come first?*

Katy: *Maybe he stopped to eat after he had finished*

James: *We know horses are fast.*

Katy: *Yes it says it stopped to eat, which means it probably came last.*

Andrew: *Yeah, but we don't know that.*

James: *And we know horses are fast.*

Andrew: *Yeah, but horse has stopped to eat. It probably means that he [the brown horse] came second and cow came first. I think it went like this. (Andrew grabbed the cow and the brown horse.)*

Katy: *So do I, because he's fast. Horses are fast.*

James: *(Replying to Andrew.) Because he's stopped to eat.*

Again, Andrew pointed out that any surmise needed to be consistent with the clues. He suggested that Katy's conjecture that brown horse must come last extrapolated beyond what was actually stated in the clues.

The clue 'The white horse saw four legs beat him home' was another focus for comment. The children queried the value of this information given that all animals in the race had four legs.

James: *Read yours again Andrew, read your clue again.*

Andrew: *Goat wanted to bite mother pig's tail as he followed her across the finish line. The white horse saw four legs beat him.*

Cam: *What's your first clue again?*

James & Andrew: *(In unison reading the clue.) The white horse saw four legs beat him home.*

Katy: *They all have four legs though.*

Cam: *All of them have four legs. That's stupid.*

Katy: *Yeah, that's true.*

The children and the plastic animal manipulatives

The children's personal commitment to the brown horse appeared to influence their preference for a finishing order. The placement of the brown horse (not the white one) was a focus of discussion. When the brown horse was first produced, James, then Katy reached for it. James stated, *I bet the brown horse wins*. The particular interest these two children displayed in the brown horse persisted throughout the time they worked on the problem. They remained wedded to the idea that horses are fast runners and the brown horse would win the race. They accorded status to their experience of and feelings towards the brown horse. Late in the solution process, they lamented:

James: *I don't think a horse could be last.*

Katy: *I can't believe my favorite animal [the brown horse] came last. This is **really** hard.*

The children and the social dimensions of working on the task

The four children worked cooperatively on the task. They each read and re-read the clues, sometimes in response to a request and once in unison, as they sought to solve the problem by reconciling the provided information. They were careful to avoid conflict. On one occasion James was unable to persuade the others that the cow was slow and so could not be first. He suggested a vote as a means to resolve the issue.

Cam: *Gosh. So that means that it is going to be either like that or like that. (Cam moved the cow and the horse so their positions were swapped on the placement mat, then moved them back again.)*

Andrew: *I think we had it right before.*

Katy: *Who was right?*

James: *Everybody's right.*

Cam: *No, he [the brown horse] stops to eat.*

James: *Hey everybody, we'll have a vote, have a vote then.*

Andrew: *No, that doesn't solve anything then James.*

James: *Yeah, but there's no point arguing then. Right, who votes for the horse to be last and the cow to be first?* (The other three children put their hands up.)

The vote was three to one for the cow being first. James accepted the outcome without demur. Here it appeared James called on a social norm of voting to invoke a process of social decision making. The action of voting and the consequence of it was taken as shared by all four children.

Children's purposes while doing the tasks

The children gave consideration to whether their agreed solution might be 'right'. Once, Andrew stated, *I think we've got it right*. The other children took up this call and shortly after the administrator asked the group if they had reached a consensus.

Andrew: *I think we've got it right.*

James: *Yeah, we've got it.*

Administrator: *All right, when you think you've finished, I'll call out the name of the animal.*

As soon as the administrator had recorded the order of the animals James asked if the recorded order was right. The administrator responded that she did not know.

The task required the administrator to ask the children to check 'one more time' to make sure their solution fitted with all the clues. The children responded by saying they were finished and there was no point in *arguing about it* [the solution]. The administrator responded, *I've asked you to argue about it*. They began the task anew, their comments and actions suggesting they interpreted the request to check the answer as a signal that their first solution was incorrect.

4.2 Group 2 and the *Farmyard Race*

Group 2 comprised of two boys and two girls. They were seated around one corner of a rectangular table; Anna was on the left of the video camera, Sarah was around the corner from Anna. Joe and Charles were beside Sarah on the long side of the table.

Doing the task

In this group, Anna assumed the role of leader. She clarified the purpose of the task, asked the administrator if they could read the clues aloud, and she asked the children to read their clues and determined when they were to be re-read. She took overall control of the movement of the plastic animals and decided when the task was complete. The other children seemed comfortable with Anna's management of the resolution process.

The group spent four and half minutes on their first attempt and two minutes on their second attempt. Anna seemed to appreciate the task as a cooperative one but the group did not produce a correct solution to it.

Their first solution was to place the goat first, white horse second, dog third, mother pig fourth, cow fifth, sheep sixth, piglet seventh and brown eighth. For the second solution they placed the animals from first to eighth in the following way: sheep, white horse, dog, mother pig, goat, cow, piglet and brown horse.

The children and the context

There was no dialogue in which the children talked about the relationships between the context of the task and their everyday experiences.

The children and the language and mathematical ideas embedded in the task

Almost immediately, Anna began to orchestrate the solution of the task. Anna requested information from each child as she sought to determine the animals' position on the basis of the clues. She positioned and re-positioned the plastic animals with very little reference to the other children to elicit their thinking.

Anna: *She's first. She's second. She must be third.*
Joe: *Yeah.*
Anna: *OK what does yours say, Joe?*
Anna: *OK. I'll read mine again.*
Anna: *Dog was third to cross the finish line.*
Anna: *So it must be second. White horse must be second.*
Anna: *OK. Read your clue again.*
Anna: *OK, so that's right I reckon. And the dog is third*
Sarah: *It must be.* (Picked up an animal, not obvious which one it was)
Anna: *No.*
Charles: *No.* (Moving dog back, suggesting Sarah had moved it).

In this episode, Charles acted to ensure that the dog retained third place. Charles placed the dog in third place on his first reading of the clue. Throughout the solution process Charles acted to preserve the dog's position. He stopped Sarah from shifting the dog from third place saying, *No, you can't move that. The dog is definitely third.* He later pointed out to the administrator when she was recording their solution, that there was a clue that *even said* where the dog came.

Part way through the first solution attempt Anna began to interact with suggestions from the other children beyond eliciting information from the clues. Anna, Charles and Joe were proficient in the use of ordinal terms and were able to make inferences on the basis of the information. In following sequence Anna, Joe and Charles concurred that the brown *would have* come eighth as a consequence of it stopping to eat before the finish line.

Anna: *Piglet finished second to last. Piglet is seventh.* (Moved piglet along the mat)
Read yours again.
Joe: *Oh.* (Went to move an animal)
Anna: (Anna interrupted Joe). *He must be eighth. Led the pack.*
Joe: *No. He would have been first.*
Anna: *But he stopped to eat.*
Charles: *So he would have been eighth.*
Anna: *He would have stopped so* (looking at animals). *That's right.*

Anna maintained her focus on the information in the clues. In the next sequence she asked to read other children's clues and demanded that they provide evidence in support of their claims about animal position.

- Anna: *So mother pig must be there. Must be fourth. And the goat might be first.*
Charles: (Referring to the dog) *He's definitely third.*
Anna: *So who says's um, um, the white horse.*
Can I read it? (Grabbing card from Sarah)
Charles: *No, my one says the dog.*
Charles: *No, my one says the dog.*
Anna: *It's your one (Grabbing Joe's card). Horse saw four legs beat him home. I reckon it's the goat [is first].*
Charles: *The sheep and piglet ran together so he'd be fifth (It was not obvious which animal was being referred to).*
Anna: *Who says that? Where does it say that?*
Charles: *It says sheep and piglet ran together, until the end when piglet tipped, tripped, so it would be there (Pointing to fifth place on the placement mat).*

Then, with no consultation Anna asserted that the task was complete.

- Anna: *Yeah. Done it!*

It was surprising that when the administrator asked the children, *Who came first?* Charles quickly answered, *Brown horse, but he stopped to eat* although the brown horse was placed last on mat. It was not clear why he said this; perhaps he responded this way by referring only to the first part of the clue concerning the placement of brown horse.

The children and the plastic animal manipulatives

Anna, more so than the other children, moved the animals to track her thinking about the finishing order. The other children referred by pointing to the positioning when they queried a placement. All of the children referred to the plastic animals as if they were objects, always linking them to the written clue.

The children and the social dimensions of working on the task

In this group, Anna assumed the role of leader right from the start and the other children accepted her direction without question. She clarified with the facilitator whether the group was expected to read their clues out loud. She asked group members in turn to read aloud their clue card. As leader, Anna also took control of the plastic animals. She moved them in relation to what she was hearing, although the other children did move animals from time to time.

There was every indication that Anna understood the function of the clues was to impose an outcome. When confirming the order of the animals she guided the group to review the set of clues by asking each individual to re-read their clues.

- Anna: *We've done that. [listening to the clues]*
Anna: (Looking at Joe) *What does yours say?*
(Joe read his clue)
Anna: *Hang on, just start again.*
(She listened to the clue and checked the placements)
Anna: *You go, Joe.*

(Joe read a clue. Anna was looking at the order of the animals at this point)

Anna: *Can I read it?* [the clue card]

(Anna reached across and took Joe's clue)

Anna: *So, that's right, I reckon.* (Looking at the animals)

The children's purposes while doing the task

The children seemed to be aware of the need to satisfy the clue conditions. The group, led by Anna, seemed to be eager to form a rapid conclusion.

4.2 Group 3 and the *Farmyard Race*

Group 3 comprised of four girls (Kate, Cherie, Gail, Georgia). They were seated around the short end of a rectangular table; one child on each end and two directly in front of the video camera.

Doing the task

This group completed their first attempt of the task very quickly, much to the surprise of the administrator. The girls each read their clues and moved the animals independently, without consideration of information in each other's clues. It appeared that they did not fully appreciate the function of the clues in specifying where the animals should be placed. It was not apparent how the group reached a consensus for the first solution. During the second attempt there was more consideration of other clues but the group did not produce a correct solution to the task.

The group spent one minute forty-five seconds on their first attempt and three minutes on their second attempt. Their first solution was made by putting the brown horse first, cow second, dog third, sheep fourth, piglet fifth, white horse sixth, goat seventh and mother pig eighth. For the second solution the animals were placed from first to eighth this way: white horse, sheep, dog, cow, mother pig, goat, piglet, brown horse.

The children and the context

The children did not mention past experiences when solving the task.

The children and the language and mathematical ideas embedded in the task

Given that there was minimal verbal interaction it was difficult to identify how the girls solved the problem in the first instance. Cherie initiated the placement process before any clue was read. She put the white horse into first place, asserting *White horse is first*. Kate silently read her clue and then stated, *OK. Dog was third to cross the finish line*. She moved dog into third position and instructed, *Read the second clue*. Cherie stood up and said *Brown horse led the pack* [the first part of the clue]. She moved the brown horse to first place on the mat. Then, without being prompted, Kate read aloud, *Sheep and piglet ran together until the end when piglet tripped*. She asked herself, *Where's sheep?* She picked up sheep. Georgia repeated, *Piglet tripped* and

moved it (it was not possible to see where she placed the animal). Georgia read her own clue, *The white horse saw four legs beat him home. The white horse saw four legs beat him home* but did not take any action.

The children continued murmuring to themselves, obviously reading their clues then moving the animals. In the first attempt there was little consultation and no overt checking of the information in each other's clues. Kate and Georgia had some dialogue about the relative positioning of goat and mother pig. Then Kate announced that the group had finished. In all, the group took less than two minutes to come to this point.

Georgia: *Goat wanted to bite mother pig's tail.*
Kate: *Wanted to bite. So, he'll be in front.*
Georgia: *Wanted to bite. Wanted to bite. So he's probably. Oh no.*
Kate: *Wanted to. OK we've finished. Aren't we?*

After the administrator recorded the finishing places of the animals for the first attempt, she emphasised that the girls needed to redo the task by reading the clues and listening to them all. When asked to check their solution the children did this willingly. Kate seemed appreciate the nature of the task a little more clearly and took responsibility for ensuring that all clues were read.

Kate: *Sheep and piglet ran together until the end when piglet tripped. So sheep and piglet were running together.*
Gail: *At the end. This.*
Cherie: *This must be first, second, third, fourth, fifth, sixth.*
Kate: *Oh, oh, oh, oh.*
Gail: *(Silently moved two animals.)*
Kate: *Running together. So dog, we know that. OK, read your clue out.*
Georgia: *So read yours out. (Pointing to Cherie.)*

The girls began to use the clues focusing on the positioning words "between" and "until" and using them to inform the positioning of the animals on the placement mat. They worked more collaboratively. In the following sequence the placement mat and animals served as referent for the dialogue that ensued.

Kate: *So two animals were between.*
Cherie: *No, two animals were between mother pig and her piglet*
Kate: *OK between them. So that will be there.*
Georgia: *The horse will go there. (Moving the white horse to the front.)*
Kate: *No, the cow will go there. Listen. Listen to mine though. Sheep and piglet ran together until the end when piglet tripped. So that's the end. Yeah. So we can put the mother pig. And the dog's in between OK? (Looking at Cherie)*
Gail: *(Moving animals silently)*

Gail, who had been participating mostly by moving animals, intervened when Kate went to move the goat. She said, *No, don't put that there. Don't put the goat there because you don't know where it finished. Piglet finished second to last* (moving piglet). Kate responded by requesting more information. Gail followed up by reading out her clues and asking Georgia to read hers again. Again, Kate terminated the conversation abruptly by simply stating they had *Got it*. In this second solution only the dog remained in the same place, perhaps because of Kate's certainty that there was only one meaning for that clue.

The children and the plastic animal manipulatives

During the first solution attempt the children appeared to work independently with the plastic animals, moving them to meet the perceived requirements of their particular clues. During their second attempt, there was more negotiation and greater collective agreement about the moving of the animals.

The children and the social dimensions of working on the task

The children in Group 3 appeared to have no appreciation of the cooperative dimension of the task. They sat together amiably but did not really interact with each other. On the second attempt Kate did attempt to manage the reading out of the clues but there was no systematic reading of, or listening to the clues as set. On several occasions a reader read only one of the clues on their clue card. Kate stated at one point, *Don't put that there. You don't know that*, suggesting she was thinking about the meaning of other clues but she did not follow up her comment to help the group reach a consensus solution.

The children's purposes while doing the task

Initially, the children seemed unsure about their expected role as collaborators and communicators in a 6-bit cooperative logic problem. In the second attempt, the children worked more collaboratively to utilise the clues. However, on both attempts, Kate arbitrarily decided the task was concluded.

4.4 Group 4 and the *Farmyard Race*

It was difficult to judge the composition of Group 4: there were two boys and one girl and one child whose gender we could not identify. The children were seated in a U shape around a rectangular table. Simon was side on to the camera, Sally sat on the left hand side of the table, Michael and the other child, (Kelly) sat facing the camera. Simon was sometimes excluded from view.

Doing the task

This group appeared to be self-conscious in front of the camera. It was often quite difficult to hear what they were saying. Michael whispered throughout the task; Sally was inclined to giggle. The administrator reminded the children to behave. Sometimes the children seemed to be attending to the clues and thinking about them as contributing to the overall solution to the tasks. At other times their attention wandered.

The group did not produce a correct solution to the task. Their first solution was to place the brown horse first, cow second, dog third, piglet fourth, white horse fifth, sheep sixth, mother pig seventh and the goat eighth. Their second solution with placed the animals from first to eighth was: brown horse. cow, dog. mother pig, sheep, goat, white horse, piglet.

The group spent three minutes on their first attempt and two minutes twenty seconds on their second attempt.

The children and the context

These children did not invoke their everyday knowledge and experience when attempting to solve the task.

The children and the language and mathematical ideas embedded in the task

As the administrator put out the animals Michael and Kelly grabbed at some of the animals and walked them along the mat. Once the task had been introduced Michael, who had dog clue, positioned the dog. Michael also positioned piglet on the placement mat. He did not confer with the other children about the content of his clues, and his positioning of animals was not challenged by the other children.

The children did not systematically read aloud the clues but Michael tried to get the group to focus on the content of them. He pointed out that his clue said, *Until piglet tripped, so that means that sheep came **before***. Later, pointing to the placement mat he reiterated, *The sheep should be right there. The sheep must be [there]* He asked, Kelly, *What does yours say?*

The children made some use of the clues when they were read. Kelly said, *That means the goat must be there* presumably as a response to the clue *Goat wanted to bite mother pig's tail as he followed her across the finish line*. Kelly disagreed with a repositioning of the mother pig and the piglet saying, *No, there's meant to be two between them*.

After three minutes, the administrator approached the children and asked them if they were finished. They appeared to be surprised by the interruption. Michael replied that the group had an order and the administrator recorded it. As she was doing this Michael told the administrator he was not satisfied with the solution.

During their second attempt the children re-read the clues and changed the position of some animals, but not the dog or the piglet. In response to a query from the administrator they said they were *happy* with their second solution although Michael seemed to defer to Sally and Simon by appearing to reluctantly agree.

The children and the plastic animal manipulatives

Initially, Kelly and Michael showed special interest in the plastic animals. Kelly made a grab for the brown horse as the administrator placed it on the table. Michael picked up the dog as it was placed down. The children's attention to these animals did not translate into any particular long-term commitment to the finishing position of either animal.

The children and the social dimensions of working on the task

The group solution process raised questions about the impact of student social and academic identity on solution processes and, ultimately on the group solution. Only one child, Michael, appeared to recognise the task was a cooperative logic one. He tried to get the other group members to think about each other's clues

and their implications for the positioning of the animals their clues referred to. Even though his reasoning about the clues was logical he was not able to convince the others of the merit of his ideas. At one stage when Simon went to move the dog from the third place position, he said *But look it says here on this card that the dog was third to cross the finish line*, Simon moved the dog anyway. Michael made three attempts to get the other children to appreciate the need to deduce the animal placement order using the information in the clues. He did not succeed in accessing group support for his suggestions. Since Michael whispered his contributions perhaps this had the effect of diminishing his authority. However, it is also possible that Michael's identity within the wider class was not one of established authority or credibility.

The children's purposes while doing the task

This group appeared somewhat disconcerted by their involvement in the NEMP assessment program. Twice, the administrator reminded them to focus on the task. Kelly waved at the camera. Three of the children seemed eager to bring the task to a conclusion.

4.5 Group 5 and the *Farmyard Race*

Group 5 comprised of three girls and one boy. The children were seated around the long side of a rectangular table: Gary on the left, Emma on the right and Teresa and Toni at the back. Sometimes it was difficult to see Gary.

Doing the task

The four children worked together to solve the task but only through the persistence of Emma to be involved. The length of the table was such that she was excluded from easy access to the manipulatives. To counter this she leaned over and rested on the table. When instructed by one of the children, *Emma, put your feet on the ground* she replied, *No, I can't see very well*. This action granted her social, intellectual and physical access to the task.

The group spent two minutes on their first attempt and one minute on their second attempt. They did not produce a correct solution. Their first solution was to place the brown horse first, white horse second, dog third, mother pig fourth, cow fifth, sheep sixth, piglet seventh and goat eighth. For the second solution they placed the animals from first to eighth as follows: brown horse, white horse, dog, cow, mother pig sheep, piglet and goat.

The children and the context

The children did not question the context of the task.

The children and the language and mathematical ideas embedded in the task

After being left to begin the task, Gary took hold of the brown horse again and placed it first. Emma grabbed the dog, and put it in the third position. Prior to this she had been studying her clue card. Emma was assertive

in overseeing that the dog was not moved from its place in the third box on the placement mat. She called upon the clue as a source of her authority, declaring, *No, don't move the dog. It tells me on my card.* It seemed as though she had a real sense of ownership of the dog and its position.

The children did not systematically read out and or combine the clues although individuals checked neighbour's clue cards.

Emma: *In this one, sheep came **first**, It says there. It says here on my card.*
Toni: *(Picking up the white horse and moving it back to the position it had been.)*
Gary: *No, it doesn't.*
Toni: *Yes it does, it stays here. It tells me on my card.*
Emma: *Can I read your card? (Gary grabbed at Emma's clue card.) The white horse finished before the goat. I think this one is behind this one. Done it.*

The children seemed to be eager to conclude the task. They were also concerned to obtain the correct answer but the way they called the administrator to them once they had placed all the animals suggested they had no real understanding of the task as a cooperative logic one.

Emma: *Done it! We hope it's right.*
(The administrator came over)
Administrator: *How did you decide?*
Emma: *We read the cards.*
Administrator: *OK you used these cards to find out which order they [the animals] finished in?*
Emma: *Are they in the right order?*

The administrator recorded their order and emphasized the group needed to *Look at **each** clue. Make sure and **then** check.*

The children concluded the review process rapidly with some checking of the clues. Gary tapped Emma on the shoulder to draw her attention to his clue. He said, *It says on this card two animals came between mother pig and the piglet. **Two**.* Emma, moving the pig and piglet said, *That means he has to move up here and he has to go there.* Gary agreed with her placement saying, *Yep, go and get [the administrator].* When the administrator asked, *Are you **all** quite sure?* The children assured him they were.

The children and the plastic animal manipulatives

As the administrator put out the animals he asked the children what they were. The children identified them as they were put down on the table, although Emma initially argued for the white horse to be called a donkey. Later, Teresa checked with the administrator, *Is this (pointing to the white horse) a horse?* When they heard their task was to determine who came first, Teresa and Gary both pointed to the brown horse intimating that they expected it to be first. The brown horse was placed first in both group solutions.

The children and the social dimensions of working on the task

It was apparent that the orientation of the table meant only three children (those around a corner) could work easily together. Emma seemed to appreciate this and quickly moved so that she was included in the

negotiation with the other children. Most of the discussion was between Emma and Gary further reaffirming Emma's desire to be involved with the solution of the task. Sometimes, however, she seemed to work independently. For instance, as she read aloud the clue 'Sheep and piglet ran together until the end when piglet tripped' she moved the animals without any reference to the other children.

The children's purposes while doing the task

The way the group quickly called the administrator to them once they had placed all the animals suggested they had no real understanding of the task as a cooperative logic one, although they expressed concern about being right. The administrator asked them how they had decided on the order, Emma responded, *We got these cards*. As the administrator recorded the order Emma said, *Hope it is right*. The administrator asked the children to review the task, emphasizing the group needed to *Look at each clue. Make sure and then check*. The children continued much as before, finishing rapidly with no systematic checking of the clues. They seemed to be eager to conclude the task.

4.6 Group 6 and the *Farmyard Race*

Group 6 comprised of three boys and one girl. The children were seated around a rectangular table: Alice on the left, Tama on the right and Hemi and Tane at the back.

Doing the task

The three boys worked cooperatively to solve the task. Alice sat completely still for most of the time, an onlooker who appeared interested in that she looked at her clue card and watched what the boys were doing. She did not speak throughout either solution process.

Tane adopted the role of leader and orchestrated the boys' use of information from the clue cards during the solution process. He appeared to understand the need to synthesise the information from each of the clues in conjunction with moving the plastic animals into a possible finishing order. Each of the boys read and re-read their clue and Hemi read aloud Alice's at Tane's request. The boys worked together to move the animals as they systematically revisited the information on the clue cards

The group spent nine minutes on their first attempt and two minutes 45 seconds on their second attempt. They remained on task throughout these times. They produced a correct solution on their second attempt. Their first solution attempt was to place the sheep first, white horse second, dog third, mother pig fourth, goat fifth, cow sixth, piglet seventh and brown horse eighth. For the second solution they placed the animals from first to eighth as follows: cow, white horse, dog, mother pig, goat, sheep, piglet and brown horse.

The children and the context

The children were not overly influenced by the contrived nature of the task although there some discussion about whether it was possible for particular animals to finish before others. When this was questioned Tane referred the children to the story of the tortoise and the hare.

Hemi (picked up cow and put it into first place).
Tama: *I don't think a cow would beat a horse.*
Hemi: *Sometime cows can.*
Tane: *Then how would a tortoise beat a rabbit?*
Tama: *Oh yeah. We'll just have it like that.*

The children and the language and mathematical ideas embedded in the task

The solution process used by the three boys was characterised by a concern to meet the requirement of individual clues whilst satisfying their combined intent. Tane initiated and then sustained this focus through out the whole task. Hemi's first action was to show Tane his clue card. Tane read it silently, then leaned to his left and read Tama's clue card out loud.

Tane: *The white horse saw four legs beat him home. The goat wanted to bite mother pig's tail. (He then read his own clue card). The brown horse led the pack until he stopped to eat. So the mother. So. (Tane stood up to read Hemi's card again). Piglet finished second to last.*
Tama: *Must have been the brown horse.*
Tane: *No, the brown horse came last. Yours is the white horse.*
Tama: *Oh yeah. I'm the white horse*
Hemi: *Piglet tripped*
Tane: *(Standing up, leaning towards Alice) Look what it says on Alice's card. Piglet finished second to last. White horse finished before the goat.*

The boys, in particular Tane, appeared to appreciate the implications of specific words for the finishing order of the animals. The boys identified one of the clues prescribed the dog's final placement. Tane noted:

Tane: *No he's third. He's third. It says on the card.*

They also realised that piglet's place was fixed.

Tane: *(Leaning sideways to read Hemi's card). Sheep and piglet ran together until the end when piglet tripped.*
Hemi: *So he must go.*
Tama: *No? (inaudible)*
...
Tane: *But it says piglet came second to last.*
Hemi: *(Picking up piglet). This one is second to last.*

Tane observed that there was no specific information pertaining to the cow.

Tane: *It must have been the cow. It hasn't got anything about the cow on this.*
Tame: *Yeah. What about the cow?*
Tane: *Where does this go?*
Hemi: *(Picking up an animal). This one goes before the white horse.*
Tane: *This one goes before the white horse. Cow must have come first because he was. Ooh, that's strange (The indication was that he was referring to the lack of information about the cow)*

Tane was explicit about the implications of ‘until’ on the placement of the brown horse, picking it up and asserting, *No, it didn’t lead the pack. That was just once.* Hemi focused attention on the implications of only four legs crossing the finish line before white horse, wondering, *What four legs could it be?* Tane monitored the placement of animals relative to the clue information. He recommended changes to the order of the animals by directing attention to relevant clue information. This strategy had the effect of ensuring that the three boys continued to be actively engaged in considering the finishing order. The boys’ process was collaborative although strongly guided by Tane.

- Tama: *Goat there* (Tama placed the goat on the placement mat but the positioning was not clear)
- Tane: *Now wait a minute. Wait a minute. Two animals were between mother pig and her piglet. Mother pig. Two animals. Not one.* (Tane held up one finger)
- (Hemi and Tama stood up and moved animals up and down the placement mat. They both sat down.)
- Tama: *That’s better*
- Tane: *Aha* (Tane stood up again). *Let’s have another look.* (Tane read Tama’s card). *The white horse saw four legs beat him home*
- Tama: (He moved some animals but it was not possible to see which ones). *So it must be the dog.*
- Tane: *True.* (Tane read Tama’s card) *Goat wanted to bite mother pig’s tail as he followed her across the finish line.*
- (Tama moved the animals again.)

Later on during the solution process Tane was visibly struggling to reconcile the discrete clues.

- Tama: *Wait, look. The white horse saw four legs beat him home, so the goat must have beaten the white horse.*
- Tane: *But the goat was behind the white horse. It says so somewhere. See the white horse saw. Oh no, where is it?* (Tane checked the other clue cards).

He indicated the need to combine and reconcile the clues posed the major challenge in the task because of possible permutations.

- Tama: *The pig’s tail.*
- Tane: *The white horse finished before the goat.*
- Tama: (Tama was moving animals.) *The goat wanted to bite mother pig’s tail as he followed her across the finish line. So he’s supposed to be behind the pig.*
- Tane: (Tane put his hands on hips and watched what was happening then began to move the animals.) *This is so hard. This can be behind two things.*

As the solution process progressed Tane gave the impression that he coming to grips with this challenge. On their second, under Tane’s guidance, attempt this group produced a correct finishing order.

The children and the plastic animal manipulatives

The plastic animals were integral to the children’s solution process. The following excerpt illustrates the way the boys used the animals as tools to record their taken-as-shared thinking. The animals served as a dynamic ‘tool for social thinking’ (Roth and Roychoudhury, 1994), that allowed the boys to work simultaneously on the task. Using the animals and the clues the boys worked together to generate, edit and correct their ideas about the finishing order for the race.

Tane: *It doesn't say anything about that, OK? (Picking up animal – possibly the sheep - and taking it off the mat.) Goat wanted to bite mother pig's tail, so he must have been there.*

Tama: *And the sheep. (Moving the animals)*

Tane: *(Moving animals.) It's either the sheep or the horse. He didn't. He didn't. (Picking up brown horse.) He was the leader of the pack until he stopped to eat.*

Hemi: *He must be.*

Tama: *Wrong. These are the other way round (Tama moved the dog back into third place.) That's better.*

Tane: *That's it, that's it. That's it.*

The children and the social dimensions of working on the task

It was striking that the three boys worked on the task as a cooperative logic one. Tane, in particular, used a range of strategies to involve the three other children in the solution process. Despite Alice's lack of active participation he solicited information from her. When nothing was forthcoming he simply asked Hemi, who was sitting closest to her, to read the information on her clue card.

Tama: *Must be cow, not sheep (Moving animals)*

Hemi: *Saw four legs beat him across the finish line.*

Tane: *What do you say Alice?*
 (Alice pulled her card towards herself and appeared to read it, but did not say anything.)

Hemi: *(Reading Alice's card). The white horse finished before the goat.*

Throughout the solution process, each of the boys evaluated the evolving placement order, treating it as a group construct. The boys often moved the animals without recourse to each other's opinion. Their evaluation revolved around the connection between the information on the clues and the animal placement. There was no evidence of personal sensitivity towards having an animal position altered.

Hemi: *(Standing up, moving animals). This should be there, and this should be there.*

Tane: *No, this has to go here like this.*

Tama: *That's better.*

Tane: *I like, I like how we had it last time. OK, the cow was first. He was on the back here (moving white horse). Who was second?*

The children's purposes while doing the task

The three boys, guided by Tane, focused their attention on producing a race finishing order that reconciled the requirements of the information on the four clue cards. Tane acknowledged the complexity of dealing with the aggregated information when this allowed for more than one possibility.

Tane: *But the goat is behind the white horse, and the dog was third to finish. This is hard.*

SECTION 5: A CROSS CASE SYNTHESIS

The following table synthesises the group responses across the dimensions of context, language and the mathematics embedded in the task, the use of manipulatives, patterns of social interaction and purposes for task completion.

| Category | Cross-case themes |
|----------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| The context | |
| Commonalities among the groups/children | Groups 2 to 6 seemed to appreciate the task was a pseudo-real/contrived one. They tackled it as a school mathematics task and did not call on their everyday knowledge of the speed of animals to help solve the task. |
| Variation among the groups/ children | <p>For Group 1, past experience was influential. James and Katy, in particular, attempted to use their knowledge of the speed of animals to determine the placement of the animals, sometimes to the detriment to their solving the task as a mathematical logic problem.</p> <p>Andrew in Group 1 indicated he was aware the task might be a ‘pretend’ one.</p> <p>Tane in Group 6 questioned the authenticity of the animal placings in the race, but reconciled this with reference to the tortoise and the hare story.</p> |
| Language and the mathematics embedded in the task | |
| Commonalities among the groups/children | <p>The reading of the clues did not appear to be an issue.</p> <p>All of the groups recognised that the clue about the dog was prescriptive – <i>Dog was third to cross the finish line.</i></p> <p>Many children did not make sense of the information that <i>Piglet finished second to last</i> in the same way, perhaps because piglet’s placement was also circumscribed by the need for it to be near sheep (<i>Sheep and piglet ran together until the end when piglet tripped.</i>)</p> <p>There was evidence that the children appreciated the implications of the ordinal language.</p> <p>Four groups debated the meaning of the brown horse leading the pack ‘until’ he stopped to eat. There was less focus on consequences of sheep and piglet running together ‘<i>until the end when piglet tripped</i>’. Four groups discussed the significance of the word ‘before’ in the clue, <i>The white horse finished before the goat.</i></p> <p>The children struggled to synthesise information from different clues to infer a placement that reconciled more than one option. The blending together of information posed a significant challenge to all groups.</p> |

| | |
|--------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Variation among the groups/children | |
| The children's use of manipulatives | |
| Commonalities among the groups | <p>The placement mat, clue cards and the plastic animals served as a taken-as-shared problem space.</p> <p>Resources served as a means of organising the task, a problem space for its solution and a final product. They provided a reference point for collaboration to make sense of / reconcile the information.</p> <p>Sometimes children read each other's clue cards.</p> <p>The children pointed, gestured, and physically manipulated the plastic animals to elaborate their talk.</p> <p>The animals served as a moveable recording device.</p> <p>When the position of animals was changed in relation to each other, they the current thinking was fixed in the moment.</p> <p>Groups 1 and 6 used the animals as an an individual thinking tool and an interactive collaborative tool.</p> <p>Group 6 was notable in the way the three boys used the manipulatives as a group thinking tool, moving them freely up and down the placement chart as they tried to make collective, public sense of the clues. It seemed that they provisionally accepted placements with the knowledge there would be an opportunity to return to previous placements at a later time.</p> <p>In Group 1 all of the children moved the animals, frequently in response to each other's suggestions as they sought find a group solution.</p> <p>Holder of the clue about the dog acted to maintain its position.</p> |
| Variation among the groups | <p>The brown and white horse seemed to have intrinsic appeal for some children. They appeared to want one of the horses to come first, irregardless of the clues.</p> <p>In Group 2 Anna as self appointed leader controlled and manipulated the animals in the main.</p> <p>Group 3 used the animals to work on the task simultaneously but not collaboratively. Initially, they read their own clues silently and moved the animals without consultation.</p> <p>When looking at where to place the cow, Tane in Group 6 commented there was no mention of the cow in the clues.</p> |

| The children and the social dimensions of working on the task | |
|----------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Commonalities among the groups/ children | <p>All the groups/children worked amicably.</p> <p>The physical arrangement influenced access to the placement mat and plastic animals as a shared problem space.</p> <p>In all six groups the three children who were positioned around a corner had easy access to these resources. The positioning of fourth child was problematic, partly because of the position of the video camera.</p> |
| Variation among the groups/children | <p>Sometimes children worked simultaneously, not cooperatively on the task. They did not appreciate the meaning of the administrator's verbal instruction <i>your team is going to put the animals in the order that they came in the race</i>.</p> <p>Neither did they appreciate the significance of the directive <i>go back and check one more time to make sure that what you've done fits with all of the clues</i>.</p> <p>All six groups responded to this directive by revisiting their solution.</p> <p>A leader of sorts emerged in Groups 2, 3 and 6 who shaped the group appreciation and management of the task.</p> <p>Group 2: Anna took charge from the beginning. She orchestrated the solution process, moved the animals mostly, requested others to read their clues and checked whether the animal placement met the requirements of particular clues.</p> <p>Group 3: Kate emerged as a leader during the second solution process. She asked other children to pool their information.</p> <p>Group 6: Tane acted to ensure that the children worked on the task as a group. He solicited information and ideas from all group members, including Alice who did not speak throughout. The three boys each moved the animals.</p> <p>Tane invoked strategies to ensure all information was made public and accounted for within the group solution.</p> <p>In Group 4 Michael appeared to understand the task as cooperative logic one but did not have the personal or social resources to mobilise and lead the other children.</p> <p>In Group 5 and in Group 3 prior to Kate's emergence as a leader, the children moved animals in relation to their own clues and desires with little consideration of other constraints.</p> <p>Group 1 worked together democratically on the task.</p> <p>No obvious leader emerged. James called on a taken-a-shared practice, voting to expedite a group decision about the placement of the brown horse relative to the cow.</p> <p>Group 6 had a silent participant (Alice).</p> |

| Children's purposes while doing the task | |
|-------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Commonalities among the groups/ children | <p>All of the groups worked purposefully.</p> <p>Groups 2, 3, 4 and 5 appeared eager to complete the task as quickly as they could.</p> <p>Attention to the information on the clue cards varied across and within the groups.</p> <ul style="list-style-type: none"> - Group 5 and Group 3 initially gave no indication they understood the need to collaborate to address the clues as a coherent whole. - Groups 1, 2 and 5 indicated a concern to be 'right'. - The discussion of Groups 1 and 6 was concentrated on the implications of combining clues. |
| Variation between the groups | |

SECTION 6: DISCUSSION

The purpose of this study was to explore the potential of a sociocultural framework as an analytical tool for making sense of children's participation in mathematical tasks. A sociocultural framework focuses attention on the interactions, a task, associated resources; the language used, and the social understandings invoked as children work together as purposeful participants. Specifically, we were interested in extending our understanding of how the context and subject content frame the ways children respond to a mathematical task. Our analysis highlighted the multifaceted impacts on children's performance.

The *Farmyard Race* was designed as group task but in each of the groups we analysed children participated in qualitatively different ways. The children's responses emphasise that mathematical tasks have intellectual, physical, material and social-relational dimensions. It was evident that all the children had learned how to participate in social practices required for successful group work that involved activities such as reading clues and manipulating materials but that only some children could use the information to make mathematical inferences and to respond to others' ideas.

The context and its impact on children's participation in group mathematical tasks

The willingness with which the children participated in the task underlined that a context can be motivating. It can resonate with children's personal realities and provide a setting for invoking and making mathematical thinking public. The *Farmyard Race* as a contrived task raises this issue when used as a tool to render thinking visible. All the children could reasonably be expected to be familiar with each of the eight animals and with the running of races but in reality cows, horses and pigs do not race each other. As a logic problem, the task required the children to attend to the clues rather than to their everyday knowledge. A difficulty can occur when children are asked to engage with a context whilst simultaneously ignoring factors that would be pertinent in a "real-life" situation (Boaler, 1993). We found this to be the case. The children needed to tackle the task as a school mathematics one and appreciate that their everyday knowledge and experience of animals was not useful when trying to find a solution.

The language and the mathematical ideas and their impact on children's participation in group mathematical tasks

Children's understanding and expression of mathematical ideas is bound up with the language they know and use. This is as much about listening and making sense as it is about speaking and reading (Hudsen, and Bruckman, 2004). To solve the task as a logico- mathematical one, the children needed to attend to the information in the clues. A democratic approach (used by James in Group 1) or an authoritative approach (used by Anna in Group 2) to decide on the finishing order of the animals was not appropriate. The children needed to aggregate information from more than one clue to deduce the placement of some of the animals. It was aggregation, rather than the meaning of words such as 'third' and 'second to last' that posed a challenge. The children quickly resolved the placement of the dog that finished third but struggled to place piglet who

was simultaneously reported as being second to last and running with sheep until he tripped. They recognised that an animal doing something ‘until’ an event occurred had implications for its placement but the specific meaning of ‘until’ proved ambiguous in the context of the race when a clue was considered in isolation. As an assessment task that sought to elicit children’s thinking such ambiguity could be construed as problematic. However, it served to stimulate discussion. Speculative possibilities about the mathematical meaning of the language helped successful partnerships unpack the clues.

Manipulatives and their impact on children’s participation in group mathematical tasks

Manipulatives can serve as a means of organising a task, a problem space for its solution and a final product that is the outcome of the solution process (Roth and Roychoudhury, 1992). The placement mat, the clue cards and the plastic animals allowed the children to work independently yet simultaneously to develop a solution to the farmyard race task. Manipulatives also provided a taken-as-shared conceptual space in which the configuration of the placement mat and the animals interlocked with the children’s talk to form and shape multimodal communication. The placement mat was a focal artefact (Roth, McGinn, Woszczyzna and Boutonne, 1999) to assist the children to reconcile the information in the clues. It served the function of assisting the children to make decisions about the order that the animals would come in the race. The children used the placement mat as a tool for anchoring, (albeit sometimes very briefly) current thinking about the positional order of the animals. The children also used the placement mat as a mediation tool so the final order of the animals could be agreed upon. As manipulatives, the clue cards proved to be an easy source of reference when a child needed to read a clue silently or aloud. Sometimes children read each other’s clues, which enabled the cards to be a shared source of readily available information,

Concrete representational manipulatives play a central role in children’s collaborative problem solving (Voigt, 1994). They help to focus, structure and expand children’s thinking and interactions and support exploration and joint meaning making (Roth et al, 1999). The children pointed, gestured, and physically manipulated the plastic animals to elaborate their talk. The animals allowed the children to explore and manage the possibilities arising from the aggregation of the clues in a way that was accessible to all in the group. Possible finishing orders were made visible. The animals were the fixed in the moment representation of current thinking. As a moveable recording device their position in relation to each other could easily be changed.

The spatial dimension to children’s participation in group mathematical tasks

Tasks have a physical-spatial aspect (Wallace, 2004). The physical arrangement of the children around a table needs to be taken into account when children are expected to work cooperatively (Jordan & Henderson, 1995, pp. 74-75). Children’s positioning around the table influenced their access to the shared problem space constituted by the placement mat, plastic animals and group discussion. However, the space was such that children could and did move to access these resources.

The social aspects of children's participation in group mathematical tasks

Group activity has a social dimension. Group tasks rely on children deploying a range of social practices that allow them to work together. Although group work is recommended as an effective strategy for fostering children's learning and problem solving, research suggests that children working in a group have differential access to material resources, ideas and the ensuing talk (Bianchini, 1997).

The *Farmyard Race* task is a cooperative logic task. Each child had a card with two clues recorded on it that they were expected to read aloud to other group members. The mechanism of each person being given a card is intended to ensure that everyone has something to contribute to the solution process. Not all groups appeared to be familiar with the cooperative logic structure.

All groups appeared to understand what it meant to work together amicably. However, some groups were more successful than others in working collaboratively on the task as a logico-mathematical one. Some children worked simultaneously but not cooperatively on the task.

Children may not have the same social standing and skills within a group. As a consequence all children do not have the same access to talk and materials when they come to work on a group task (Bianchini, 1997). Research with adults indicates an effective leader can work with others to develop a shared vision and a way of achieving this (Fullan, 2001). When a child assumed leadership responsibility s/he had a significant impact on what the group sought to achieve and consequently what it was able to accomplish. If the children worked independently they were inclined to approach the meaning making of the clues in a haphazard way.

Children's purposes while participating in group mathematical tasks

Children are able to engage in group tasks at their own pace and in ways that are of their own choosing, although the parameters of the task are set (Sare and Guberman, 1998). Four of the groups in this study, perhaps because of the contrived nature of the environment (Goos, 1994), seemed driven to finish the task quickly. Children's purposes shift and take form as they interact about a task (Sare and Guberman, 1998). Their mathematical goals are interwoven with the context and artefacts of a task. Some children pursued goals to facilitate rapid task completion; others pursued goals to do with their favourite animals winning the race, others worked to make sense of all the clues. When the child who assumed the role of leader did not understand the interdependency of the clues, and consequently the intellectual intention of the task, its cooperative logic structure was ignored. Prior experience of this type of task may have impacted on the goals the children chose to pursue.

Commentary on the boundaries of the study

The analysis of six short sequences of children participating in a group mathematical task highlighted the need to pay attention to all of the aspects of their interaction. We were surprised at just how little time some groups worked on the task. Our initial impressions were not always borne out when we revisited the videos

to focus on differing aspects of what was happening. For us, videos added texture to the data and allowed for a more complex and comprehensive analysis.

SECTION 7: IMPLICATIONS OF A SOCIOCULTURAL INTERPRETATION OF CHILDREN'S PARTICIPATION IN A MATHEMATICS ASSESSMENT TASK

This study has explored the ways in which six groups of four children worked to find a solution to the NEMP assessment task: *The Farmyard Race*. The assessment schedule indicates that children's solution strategies and their checking and re-checking of the clues were rated on a four-point scale from 'very good' to 'poor'. The extent to which the children cooperated and whether or not one or more leaders emerged was evaluated.

We analysed videos of children's engagement with a mathematics cooperative logic task to identify how the children interacted with each other and the material and representational systems in the assessment setting. Our intention was to use a sociocultural perspective to highlight and provide some account for the diversity that emerges within and amongst groups when children participate in mathematical tasks. The children's individual involvement varied which was indicated by their attention to the manipulatives and to other children in the shared problem space (Suchman, 1988 cited in Roth and Roychoudhury, 1992).

Our analysis has highlighted the complex interaction between task context, language and manipulatives, spatial arrangements and social configurations. Children's purposes for doing and solutions to the task evolved as they used the manipulatives as tools for individual thinking and interactive multimodal communication. Their talk and the momentary representation of their thinking about the finishing order co-existed. It was difficult to make sense of the children's talk without reference to the placement mat and animal positions. Given this complexity, the record of the finishing order is inadequate to assess the product of a group deliberation. The ultimate finishing order provides no indication of how particular individuals have contributed. The process of reaching a solution provides more insight into children's thinking. For this video is invaluable, the children's processes in solving the task could not have been reconstructed without it.

The video record of the children's participation provided evidence of the children's social and leadership skills in an environment afforded by the *Farmyard Race* task. The children's actions indicated they were able to cooperate but that they were not compelled by the requirements of the task to work cooperatively. That is, the quality of the outcome was not undermined when a child did not participate. It was possible for an individual to solve the task once s/he had access to all the clues; the intent of the task could be met through individual synthesis of information. A task that has conceptual and practical demands that can only be met by the work of a group is difficult to design (Bainchini, 1997). One way to include this demand in the *Farmyard Race* task would be to require a group to justify and explain their final finishing order.

In this study, it is was not possible to determine whether children's lack of familiarity with the nature of cooperative tasks or the inability to aggregate the clues lead to their difficulties in positioning some of the animals. Presumably, the instruction to *go back and check one more time to make sure that what you've done fits with all the clues* was supposed prompt the children to consider the clues as a whole. In the six groups we analysed this instruction did not produce this effect. Other research indicates young children are reluctant to revisit a task (Moreland, 2003). In the context of school, children often interpret being asked to do something again as an indication that their first attempt was not what was required.

From a sociocultural perspective, assessment 'requires sampling across a domain of situation types in which participation involves the kinds knowing that are of interest' (Greeno, 1997). The Farmyard Race is but one of many situations in which children might be required to draw upon logico-mathematical thinking. Given the diversity in the children's solution processes and final placement orders it would seem pertinent to assess their participation in a range of situation types in which participation involves the kinds of knowing that are of interest.

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